A hybrid particle swarm optimization algorithm using adaptive learning strategy

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\textbf{A B S T R A C T}

Many optimization problems in reality have become more and more complex, which promote the research on the improvement of different optimization algorithms. The particle swarm optimization (PSO) algorithm has been proved to be an effective tool to solve various kinds of optimization problems. However, for the basic PSO, the updating strategy is mainly aims to learn the global best, and it often suffers premature convergence as well as performs poorly on many complex optimization problems, especially for multimodal problems. A hybrid PSO algorithm which employs an adaptive learning strategy (ALPSO) is developed in this paper. In ALPSO, we employ a self-learning based candidate generation strategy to ensure the exploration ability, and a competitive learning based prediction strategy to guarantee exploitation of the algorithm. To balance the exploration ability and the exploitation ability well, we design a tolerance based search direction adjustment mechanism. The experimental results on 40 benchmark test functions demonstrate that, compared with five representative PSO algorithms, ALPSO performs much better than the others in more cases, on both convergence accuracy and convergence speed.

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1. Introduction

Many real-world problems which can be transferred to the optimization problems now have become more and more complex, and are difficult to be solved by the canonical optimization algorithms. Therefore, the research on optimization algorithms in real applications has become a promising research issue. In the past few years, lots of meta-heuristic optimization algorithms have emerged \cite{4,15,24,25,29,30}. And one of the most popular algorithm is particle swarm optimization (PSO).

PSO is a nature-inspired evolutionary algorithm firstly proposed in 1995 by Kennedy et al. \cite{6}. In PSO, each solution is represented by a particle in the population with two main vectors, namely velocity and position, which are dynamically changed according to its interactions with other particles during the evolutionary process. And the particles can adjust their trajectories by their flying experiences in each generation which can help them fly towards a better search space. PSO can
be implemented easily to deal with many function optimizations with fast convergence, and in the last few decades, PSO
has been widely applied in many real optimization problems and has shown good performances, such as manufacturing
control in engineering optimization [5,16,17,22,27], and multi-source scheduling in cloud computing [9–11].

However, the basic PSO also has some drawbacks which make it get trapped in the local optimum and suffer the pre-
mature convergence, especially in the large scale complex optimization problems. Firstly, for basic PSO, the particles always
adjust their flying trajectories and converge to a single point regarding the parameter settings, and different parameter set-
tings may lead to different convergence speed. Secondly, the particles evolve according to their interactions with each other
during the search process, and the updating strategy is mainly aims to the global best particle which makes the population
lose the diversity that increases the possibilities of getting trapped in local optima.

Hence, how to improve the convergence speed as well as avert the premature convergence has become the most im-
portant research problem in PSO. In recent years, lots of research works have been done and many variants of PSO have
been put forward. These algorithms try to balance the local search ability and global search ability through the tuning of
the parameters, modification of the updating rules, designing new strategies in the evolving process and indeed get some
improvements on the convergence of the solutions. Nevertheless, with the optimization problems becoming increasingly
complex, the current algorithms still can’t guarantee a good diversity and efficiency of the solutions in reality.

In this paper, in order to overcome the above limitations, a hybrid PSO algorithm with adaptive learning based strategy
(ALPSO) is proposed, which incorporates a self-learning based candidate generation strategy to ensure exploration as well
as a competitive learning based prediction strategy to guarantee exploitation of the algorithm, and a tolerance based search
direction adjustment mechanism to well-balance exploration and exploitation.

The rest of this paper is organized as follows. A brief introduction of the standard PSO and its variants are presented in
Section 2. The details of our proposed ALPSO algorithm is described in Section 3. The experimental results and analysis on
several benchmark multimodal functions are shown in Section 4, and the conclusion is provided in Section 5.

2. PSO algorithm

2.1. Basic PSO

PSO is a swarm intelligence algorithm which was proposed based on the observation on the swarm behaviors in some
ecological system, such as birds flying, bees foraging, or social behaviors of human beings. As one of the population-based
algorithm, each particle in the population of PSO represents a possible solution of the optimization problem with two vec-
tors (velocity and position). For a problem in D-dimensional space, a particle i which with the position \( x_i = (x_{i1}, x_{i2}, \ldots, x_{id}) \) has a velocity \( v_i = (v_{i1}, v_{i2}, \ldots, v_{id}) \) when the particle is moving. During the optimization process, the velocity vector and
the position vector can be updated by the following rules:

\[
\begin{align*}
v_{id}(t + 1) &= w \cdot v_{id}(t) + c_1 \cdot r_1 \cdot (p_{id}(t) - X{id}(t)) + c_2 \cdot r_2 \cdot (P_{gd}(t) - X{id}(t)) \\
x_{id}(t + 1) &= x_{id}(t) + v_{id}(t + 1)
\end{align*}
\]

(1)

Where \( v_{id} \) denotes the velocity of the ith particle, \( x_{id} \) denotes the current position of the particle. And \( w \) denotes
the inertia factor, \( r_1 \) and \( r_2 \) are two random numbers in the interval of \([0, 1]\) which are employed to maintain the
population’s diversity, \( c_1 \) and \( c_2 \) are the accelerate coefficients which imply the relevant influence of the particles, \( P_{id} = (p_{best1}, p_{best2}, \ldots, p_{bestd}) \) is the previous best position \((p_{best})\) of the ith particle and \( P_{gd} \) is the global best particle \((g_{best})\)
found in the population.

2.2. Some variants of PSO

Due to the simplicity of implementation and efficiency of performance, PSO has become a promising tool for practical
applications which has attracted many researchers to study and improve its performance. Recently, many variants of PSO
algorithm have been proposed, which can be categorized into the following 3 types: 1) Parameter modification based PSO
algorithms, which focus on the modification or adjustment methods of the inertia weights \((w)\) and the accelerate coeffi-
cients \((c_1, c_2)\); 2) Population topology structure analysis based PSO algorithms, which try to use the information of
population topology to guide the search process; 3) Evolutionary learning strategy based PSO algorithms, which integrate
different evolutionary operators or use the historical information of the particles to design new strategies to improve the
performance. Although many kinds of PSO variants have been proposed to avoid premature convergence, some strategies
have been applied to keep the diversity of population and try to overcome the premature convergence, even some of them
sacrifice the performance on convergence speed, it is still unavoidable for the population getting trapped in the local optima
when dealing with difficult multimodal problems.

2.2.1. Parameter modification based PSO algorithms

As we mentioned above, the parameter setting has played a crucial role in PSO convergence behavior. Many research
works have shown that, if the inertia weight becomes larger, the particles’ speed will increase which will results in more
exploration and less exploitation, and vice versa. And since the particles are updated by the cogitative component and
social component, the control mechanism of these two coefficient parameters is also vital to the accuracy and efficiency of the solutions. How to find the appropriate values for these parameters is an interesting research issue.

Shi et al. took the leading role in employing a new inertia weight in PSO which can loosen the restriction on velocity and control the search process better and proposed GPSO [20]. Afterwards, different kinds of adjustment strategy on \( w \) are advocated. Some researchers defined the inertia weight as a time-varying function [1] while some others took the adaptiveness into account and adjusted the value of inertia weight adaptively by monitoring the evolutionary states [21]. Ratnaweera et al. proposed a time-varying scheme for the acceleration coefficients, which help balance the global search ability and local search ability of the algorithm [18]. Zhan et al. use the information of the population distribution and particle fitness and propose an adaptive technique which can automatically control the inertia weight, the acceleration coefficients simultaneously [28].

2.2.2. Population topology structure analysis based PSO algorithms

In the standard PSO, the population size is fixed and each particle is evolved using the information got from all other particles. As we know that a larger population size may increase the computation costs resulting in slow convergence while a smaller population size may converge in local optimum. Followed by the idea of divided-and-conquer, many researchers focus on the topology structure and proposed many multi-swarm based PSO algorithms.

Different topology structures based on the connections of neighborhood have been employed to maintain the diversity of the solution, like LPSO [7]. Liang et al. proposed a CLPSO algorithm where every particle updates its velocity by learning from the historical best information of different particles which help the particles learn more valuable information to guide their search behaviors [13]. Then, they further investigated the sub-swarm method and proposed a DMS-PSO algorithm with dynamic multi swarms, in which the population is composed of many small sub-swarms and the dynamic strategy can keep the diversity of the population at the same time [14].

2.2.3. Evolutionary learning strategy based PSO algorithms

Since different optimization algorithms might have different strength, it is reasonable to learn or combine the strategies used in these algorithms to help to design a new one. Many evolutionary operators are introduced to increase the diversity of population and help PSO jump out of the local optimum. Some evolutionary algorithms, such as genetic algorithm (GA) [3], artificial bee colony algorithm (ABC) [26], differential evolutionary algorithm (DE) [19] have been combined with PSO to improve the exploration ability.

Some other researchers investigated the flying behaviors of the particles and try to design proper learning strategies to deal with the information of the particles or swarms (sub-swarms), which are helpful for the solution jumping out of local optimum. Wang et al. designed an opposition based learning strategy where the candidates are transformed from one search space to another, which give the candidates more chance to find the global optimal solution [23]. Li et al. designed four different learning models by a self-learning approach and the particles are assigned with different role in the search process only based on their local fitness landscape [8]. Different from the traditional PSO, Cheng et al. employed the competition mechanism in which the particles are required to update their search patterns when they lose competition [2].

3. Adaptive learning based PSO algorithm (ALPSO)

As we know that all particles in original PSO learn from the global best particle to update their position and velocity until the termination condition is reached, even though the global best particle gets trapped into local optimum. This kind of learning mechanism makes the algorithm better in exploitation and have the feature of fast convergence but is invalid when dealing with the problems with complex search space. Now some representative variants of PSO are proposed, in which some strategies are used to restrict the learning object of particles to maintain the diversity of the population like LPSO, CLPSO. These strategies make the algorithm get good performance in exploitation but will slow down the convergence rate of the algorithm.

In order to balance between exploitation and exploration, the algorithm is proposed to obtain a better performance which is ALPSO. In ALPSO, we use a tolerance based search direction adjustment mechanism. The mechanism can make the swarm adjust their search direction to avoid falling into a local optimum adaptively and shrink the search space. Furthermore, we use a self-learning based candidate particle generation strategy to generate a candidate particle as a learning object of swarm, in which the swarm can do exploration in different areas within the search space. Then, for the purpose of ensuring the efficiency and accuracy of the algorithm, we use a potential prediction strategy to predict the potential ability of candidate particle to lead swarm do exploitation in different dimensions.

3.1. Tolerance based search direction adjustment mechanism (TSDM)

It is well known that the swarm in original PSO is more likely to get trapped into local optimum when searching in a complex space. For a certain dimension, the evolutionary state can be described by Fig. 1(a) where particles search for a maximum within one dimension and the red curve illustrates the variation trend of the fitness value in this dimension. \( P_g \) is the global best particle close to a local optimum. In Fig. 1(a), it is obviously that each particle, e.g., \( X_1 \) and \( X_2 \) may search along the direction to \( P_g \) and the swarm will fall into local optimum after several iterations. Suppose \( f(P_i) \) denotes the fitness value of the \( i \)th particle's best position at the \( t \)th iteration. The \( i \)th particle's current position, i.e., \( X_i \) will be compared to
Fig. 1. The evolutionary state of swarm in complex search space.

Fig. 2. Direction adjustment probability with T.

$P_i$ after each iteration, $P_i$ and $f(P_i)^t$ will be updated. The situation that all particles’ best position are not improved in one iteration can be described as Eq. (2),

$$\sum_{i=1}^{n} f(P_i)^t - f(P_i)^{t-1} = 0$$

where $n$ is the size of the population, and $t$ is the number of iteration. Obviously, the characteristic that all particles’ best position don’t get improved after one iteration may reflect that the swarm has got trapped into local optimum.

In order to avoid swarm falling into local optimum and guarantee the efficiency of algorithm, we can adjust the swarm’s search direction when Eq. (2) is satisfied. However, we can not make a decision that adjust the search direction of swarm just rely on the situation described above which emerges once especially when swarm searches in a complex solution space. As Fig. 1(b) shows, particles search for a maximum within one dimension and $P_g$ is the global best particle close to the global optimum. Particles in swarm at the $t$th iteration, e.g., $X_i^t$ and $X_j^t$ may search along the direction to $P_g$ and these particles’ best position will not be improved in the $t + 1$th iteration, and in this case the Eq. (2) is satisfied too. However, in the long-term evolution of swarm, $P_g$ has the potential ability to lead the swarm to improve and it is promising for swarm to search the global optimum in the next several iterations.

Therefore, for the purpose of avoiding that swarm gets trapped into local optimum and taking full advantage of $P_g$’s evolutionary potential, we propose a tolerance based search direction adjustment mechanism (TSDM), which can help swarm comprehend the state of evolution during the iteration of algorithm to prevent the swarm from falling into local optimum and adjust the search direction at a proper time.

In fact, as the number of occurrences of the situation that all particles’ best position are not improved increases, the probability that swarm gets trapped into local optimum will also increase, then the swarm need to adjust it’s search direc-
tion timely. We denote a variable tolerance of the swarm by $T$, and $T$ is used as a counter and initialized to 0. If all particles are not improved after one iteration, we can update the parameter $T$ as the following Eq. (3).

$$T = T + 1$$

(3)

It is obviously that, the swarm will be more likely to get into local optimum as the value of $T$ becomes larger, which means the swarm should adjust it’s search direction. We denote the probability that swarm adjust its search direction as $\text{Prob}_{\text{adjust}}$, and $\text{Prob}_{\text{adjust}}$ is empirically obtained by the following tolerance Eq. (4).

$$\text{Prob}_{\text{adjust}} = \frac{\exp(T) - 1}{\exp(10) - 1}$$

(4)

Here $\text{Prob}_{\text{adjust}}$ is updated after each iteration. Fig. 2 shows how $\text{Prob}_{\text{adjust}}$ changes with $T$. If $\text{Prob}_{\text{adjust}}$ is larger than a random number belongs to $[0, 1]$, the swarm will stop learning from the current $P_g$ and adjust it’s search direction to a new particle. Details of the procedure are shown as below in Algorithm 1.

### Algorithm 1 Tolerance based search direction adjustment mechanism.

1: Initialize $T = 0$;
2: if $\left( \sum_{i=1}^{n} f(P_i)^2 - f(P_i)^{i-1} \right) = 0$ then
3: \hspace{1cm} $T = T + 1$;  
4: end if
5: Generate a random number $\text{rand}()$ between $[0, 1]$;
6: if $\left( \frac{\exp(T) - 1}{\exp(10) - 1} \right) > \text{rand}()$ then
7: \hspace{1cm} Stop the swarm from learning the current $P_g$
8: end if

As shown in the Algorithm 1 and Fig. 2, when the tolerance value of swarm, i.e., $T$ is small, $\text{Prob}_{\text{adjust}}$ is determined by $T$ and is more likely smaller than a random number which belongs to $[0, 1]$, then swarm will still learn from current $P_g$. $P_g$’s potential leading ability will be fully utilized in the next several iterations especially when $P_g$ is close to the global optimum as shown in Fig. 1(b). If all particles are still not improved in the next several iterations, the value of $T$ will increase continuously and the swarm is more likely to fall into a local optimum, then the value of $\text{Prob}_{\text{adjust}}$ is probably larger than a random number belongs to $[0, 1]$ and the swarm will adjust its search direction by learning a new particle. The new particle is called Candidate and the detail of generating a Candidate is presented in Section 3.2. Then the swarm may jump out of the current local optimum after learning from Candidate for a period of iterations. In summary, the strategy, i.e., TSDM can take full advantage of $P_g$’s potential leading ability under the premise of helping swarm jump out of local optima.

### 3.2. Self-learning based candidate generation strategy

In ALPSO, all particle learn from $P_g$ at the beginning, swarm use the TSDM to determine whether it need to adjust its search direction. When the swarm is likely trapped into a local optimum, it will adjust its search direction by learning from a new particle which presented as Candidate. It is easy to randomly generate a Candidate in the search space and swarm can learn from the generated Candidate to jump out of the current local optimum. However, the randomly generated Candidate’s ability to lead swarm to evolve may not be guaranteed especially when swarm searches in a complex space, it is more likely that the Candidate will lead swarm into another local optimum. In order to generate the Candidate effectively, we propose a self-learning based candidate generation strategy by which the swarm learns its own advantages and makes full use of every particle’s excellent historical best structure of solution during each iteration of algorithm.

It is obviously that the current $P_g$’s fitness value is still the best, and $P_g$ still maintains good solution structure in most out of $D$ dimensions. So $P_g$’s solution structure is worth learning when generating the Candidate. In addition, because the fitness value of a particle is determined by the particle’s solution structure of all $D$ dimensions, which is represented as Eq. (5).

$$f(P_{\text{Particle}}) = f(x_1^g, x_2^g, \ldots, x_D^g)$$

(5)

Therefore, the particle whose fitness value is slightly worse may have good solution structure in some particular dimensions, and it is also worth learning under this situation.

Fig. 3 shows a evolutionary state of swarm in two certain dimensions. Particles search for a maximum in each dimension and the red curve illustrates the variation trend of fitness value in each dimension. $P_g$ is the current global best particle of swarm, $P_1$ and $P_2$ are two individual best position of $X_1$ and $X_2$, respectively. It can be seen that $P_g$ has the best solution structure in 1th dimension but $P_1$ has a little better solution structure than $P_g$ in 2nd dimension. Because the fitness value of a particle is determined by the particle’s solution structure of all $D$ dimensions, as Eq. (5) described, $P_g$ will not be replaced by $P_1$ and we may neglect the good solution structure that $P_1$ holds in 2nd dimension, which is also worth learning when generating Candidate.
For the above reasons, Candidate will be generated not only according to the $P_g$ but also the individual best solutions that all particles hold. And $\text{Candidate}(\text{candidate}^1, \text{candidate}^2, \ldots, \text{candidate}^D)$ is the Candidate particle with $D$ dimensions. The detail of generating Candidate is described as follows in Algorithm 2.

**Algorithm 2** Self-learning based candidate generation strategy.

1: for each dimension $d$ from 1 to $D$ do
2:   Generate a random number rand() between [0, 1];
3:   if $\text{Prob}_{\text{Candidate}} > \text{rand()}$ then
4:     Candidate$^d = P_g^d$;
5:   else
6:     randomly select two $P_k$ and $P_m$ from the swarm;
7:     if $f(P_k^d) < f(P_m^d)$ then
8:       Candidate$^d = P_k^d + \text{Gaussian}(\sigma^d)$;
9:     else
10:    Candidate$^d = P_m^d + \text{Gaussian}(\sigma^d)$;
11:   end if
12: end if
13: end for

Here, $\text{Prob}_{\text{Candidate}}$ is the probability between [0, 1] that Candidate$^d$ will obtain the structure of $P_g^d$ in $d$th dimension. Obviously, as the value of $\text{Prob}_{\text{Candidate}}$ increases, the generated Candidate will be more similar to $P_g$, on the contrary, the Candidate may be quite different from $P_g$ and has the ability leading swarm jump out from current local optimum. Then $\sigma^d$ is standard deviation variable which reflects the distribution of all particles’ best solution, in the swarm with $n$ particles it is obtained by the following Eqs. 6 and (7).

$$
\text{Average}^d = \frac{1}{n} \sum_{i=1}^{n} P_i^d
$$

$$
\sigma^d = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (P_i^d - \text{Average}^d)^2}
$$

Considering that the goal of generating Candidate is to lead the swarm jump out from local optima, it may be inefficient to reconstructed the solution structure of Candidate randomly within the search space. To make sure that the reconstructed solution structure is not too bad, we randomly select two particles $P_i(k)$ and $P_i(m)$, and choose the more excellent one as an example with a Gaussian offset value determined by $\sigma^d$. As the process of selecting is random, theoretically all particles in the swarm can provide their own search information to the generating of Candidate. Candidate may get a superior solution structure to the current $P_g$ and is more likely close to global optimum, so swarm may jump out of local optima by adjusting
it’s search direction to Candidate and global optimum may be found after a period of iterations. In conclusion, this strategy ensures the algorithm’s ability of exploration by utilizing all particles’ individual best solution structure and reduces the effects on the rate of convergence at the same time.

3.3. Competitive learning based prediction strategy

As described above, the Candidate is randomly generated as a new learn object. So its ability to lead the swarm can’t be guaranteed. It’s reckless for the algorithm to replace the current \( P_g \) with Candidate without any proving on the ability of Candidate. In addition, the current \( P_g \) may still have a better structure of position than Candidate which is worth learning. To enhance the exploitation of algorithm and take advantage of the current \( P_g \) and Candidate, we propose a competitive learning strategy to predict the potential leading ability of Candidate in ALPSO.

After a Candidate has been generated, ALPSO will predict the potential leading ability by evaluating the performances of the swarm in which the particles learn from the current \( P_g \) and Candidate in the next time of iteration, respectively. The velocity of these two particles will update according to the following rules.

Once the swarm learns from the current \( P_g \), the velocity will be updated by the following Eq. (8):

\[
v^d_l = \omega v^d_l + c_1 r^d_1 (P^d_l - X^d_l) + c_2 r^d_2 (P^d_g - X^d_l)
\]

(8)

And if the swarm learns from the Candidate, the velocity will be updated by the following Eq. (9):

\[
v^d_l = \omega v^d_l + c_1 r^d_1 (P^d_l - X^d_l) + c_2 r^d_2 (Candidate^d_l - X^d_l)
\]

(9)

There is a competitive relationship between the current \( P_g \) and Candidate, after one time iteration and the swarm will choose the better one as the new \( P_g \) in the next several iterations, and the worse particle will then learn from the better one to update the status of itself. This is why we name it as “competitive learning”. In order to measure the ability leading swarm of \( P_g \) and Candidate conveniently, the Eq. (10) is described as follows,

\[
Competitiveness_{P_g} = \sum_{i=1}^{n} f(x_i)^{i+1} - f(x_i)^i
\]

(10)

where \( L \) stands for the learn object of the swarm. If \( Competitiveness_{P_g} \) is larger \( Competitiveness_{Candidate} \), it means the improvement when swarm learns from the current \( P_g \) is better, and vice versa. Then swarm will choose a better one as the new \( P_g \) in the next iteration. Details of this strategy is concluded as Algorithm 3.

3.4. Framework of ALPSO algorithm

Now, we can sum up the framework of this ALPSO algorithm in the following Algorithm 4.
Table 2
The 28 CEC2013 benchmark functions.

<table>
<thead>
<tr>
<th>No.</th>
<th>Functions</th>
<th>Optimum</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>Sphere function</td>
<td>−1400</td>
</tr>
<tr>
<td>14</td>
<td>Rotated high conditioned elliptic function</td>
<td>−1300</td>
</tr>
<tr>
<td>15</td>
<td>Rotated bent cigar function</td>
<td>−1200</td>
</tr>
<tr>
<td>16</td>
<td>Rotated discus function</td>
<td>−1100</td>
</tr>
<tr>
<td>17</td>
<td>Different powers function</td>
<td>−1000</td>
</tr>
<tr>
<td>18</td>
<td>Rotated Rosenbrocks function</td>
<td>−900</td>
</tr>
<tr>
<td>19</td>
<td>Rotated Schaffers F7 function</td>
<td>−800</td>
</tr>
<tr>
<td>20</td>
<td>Rotated Ackleys function</td>
<td>−700</td>
</tr>
<tr>
<td>21</td>
<td>Rotated Weierstrass function</td>
<td>−600</td>
</tr>
<tr>
<td>22</td>
<td>Rotated Griewanks function</td>
<td>−500</td>
</tr>
<tr>
<td>23</td>
<td>Rastrigrins function</td>
<td>−400</td>
</tr>
<tr>
<td>24</td>
<td>Rotated Rastrigs function</td>
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</tr>
<tr>
<td>25</td>
<td>Non-continuous rotated Rastrigins function</td>
<td>−200</td>
</tr>
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<td>26</td>
<td>Schwefel’s function</td>
<td>−100</td>
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<tr>
<td>27</td>
<td>Rotated Schwefel’s function</td>
<td>100</td>
</tr>
<tr>
<td>28</td>
<td>Rotated Katsuura function</td>
<td>200</td>
</tr>
<tr>
<td>29</td>
<td>Lunacek-Bi-Rastrigin function</td>
<td>300</td>
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<td>30</td>
<td>Rotated Lunacek Bi-Rastrigin function</td>
<td>400</td>
</tr>
<tr>
<td>31</td>
<td>Expanded Griewanks plus Rosenbrocks function</td>
<td>500</td>
</tr>
<tr>
<td>32</td>
<td>Expanded Saffers F6 function</td>
<td>600</td>
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<td>33</td>
<td>Composition function 1 (n = 5.rotated)</td>
<td>700</td>
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<tr>
<td>34</td>
<td>Composition function 2 (n = 3.unrotated)</td>
<td>800</td>
</tr>
<tr>
<td>35</td>
<td>Composition function 3 (n = 3.rotated)</td>
<td>900</td>
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<tr>
<td>36</td>
<td>Composition function 4 (n = 3.rotated)</td>
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<td>Composition function 6 (n = 5.rotated)</td>
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<td>Composition function 7 (n = 5.rotated)</td>
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</tr>
<tr>
<td>40</td>
<td>Composition function 8 (n = 5.rotated)</td>
<td>1400</td>
</tr>
</tbody>
</table>

Algorithm 3  Competitive learning based prediction strategy.

1: for iteration from t to t + 1 do
2:   for each particle i and i from 1 to n do
3:     \text{Competitiveness}_{\text{Candidate}}^+ = f(x^i)^{t+1} - f(x^i)^t;
4:   end for
5: end for
6: for iteration from t to t + 1 do
7:   for each particle i and i from 1 to n do
8:     \text{Competitiveness}_{P_k}^+ = f(x^i)^{t+1} - f(x^i)^t;
9:   end for
10: end for
11: if \text{Competitiveness}_{\text{Candidate}} > \text{Competitiveness}_{P_k} then
12:   Update \text{Prob}_k: P_k = \text{Candidate};
13:  T = 0 // T is the tolerance counter of the swarm;
14: else
15:   \text{Prob}_k doesn’t change;
16:  T = T – 1 // T is the tolerance counter of the swarm;
17: end if

3.5. Computational complexity of ALPSO

Compared with the basic PSO, the main difference of ALPSO is the if conditional statement (from line line 9 to line 14). Obviously, the Algorithm (1) is used to update the variable T and \text{Prob}_\text{adjust}, and the computational complexity of which is only (1). In the Algorithm 2, if the D dimensions solution structure of \text{Candidate} is generated, the computational complexity of generating a \text{Candidate} is \text{O}(D). Suppose N is the size of the swarm, according to algorithm (3), which performs one iteration by applying swarm learn the \text{Prob}_k and \text{Candidate} respectively, its computational complexity is \text{O}(N \cdot D). Therefore, the if conditional statement’s computational complexity is \text{O}(N \cdot D).

As a result, if the ALPSO’s stop condition is a fixed iteration number presented as \text{Ite}, the entire computational complexity of ALPSO is \text{O}(\text{Ite} \cdot N \cdot D).
Algorithm 4 Framework of ALPSO.
1: Initialize all particles’ positions and velocities within the search space;
2: Initialize $T = 0$, $Prob_{\text{adjust}} = 0$ ;
3: Evaluate the fitness value of every particle;
4: Update $P^i$ and $P^g$ ;
5: while (stop condition in not reached) do
6: Update all particles’ $X^i_t$ and $V^i_t$;
7: Evaluate the fitness value of every particle;
8: Update $P^i$ and $P^g$ ;
9: if $P > \text{rand}()$ then
10: stop the swarm learning from the current $P_g$;
11: Use the algorithm 2 to generate a Candidate particle;
12: Use the algorithm 3 to choose a better particle as new $P_g$ from the current $P_g$ and the Candidate;
13: Use the algorithm 1 to update $T$ and $Prob_{\text{adjust}}$;
14: end if
15: end while

Table 3
Parameters settings.
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPSO</td>
<td>$\omega = 0.4$, $c_1 = c_2 = 2$</td>
</tr>
<tr>
<td>GPSO with decreasing weight</td>
<td>$\omega = [0.4, 0.9]$, $c_1 = c_2 = 2$</td>
</tr>
<tr>
<td>LPSO</td>
<td>$\chi = 0.7298$, $c_1 = c_2 = 1.49445$</td>
</tr>
<tr>
<td>DMS-PSO</td>
<td>$\chi = 0.7298$, $c_1 = c_2 = 1.49445$, $M = 4$, $R = 10$</td>
</tr>
<tr>
<td>CLPSO</td>
<td>$\omega = [0.4, 0.9]$, $c = 1.49445$, $\text{gpM} = 7$</td>
</tr>
<tr>
<td>ALPSO</td>
<td>$\omega = [0.4, 0.9]$, $c_1 = c_2 = 2$</td>
</tr>
</tbody>
</table>

Table 4
Results comparison on 12 basic functions.

<table>
<thead>
<tr>
<th>Function</th>
<th>ALPSO</th>
<th>GPSO $\omega = 0.4$</th>
<th>GPSO$\omega = [0.4, 0.9]$</th>
<th>LPSO</th>
<th>CLPSO</th>
<th>DMS-PSO</th>
</tr>
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<tr>
<td>$f_1$</td>
<td>1.70E-93</td>
<td>9.12E-162</td>
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<td>4.57E-23</td>
<td>2.02E-29</td>
<td>7.68E-28</td>
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<tr>
<td>$f_2$</td>
<td>1.24E-9</td>
<td>3.95E-161</td>
<td>3.97E-50</td>
<td>1.02E-22</td>
<td>3.70E-29</td>
<td>1.62E-27</td>
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<tr>
<td>$f_3$</td>
<td>2.13E-28</td>
<td>6.22E-91</td>
<td>2.25E-29</td>
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<td>1.68E-17</td>
<td>3.28E-13</td>
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<td>$f_4$</td>
<td>7.52E-27</td>
<td>1.89E-90</td>
<td>4.41E-29</td>
<td>9.54E-12</td>
<td>2.82E-17</td>
<td>1.24E-12</td>
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<td>$f_5$</td>
<td>6.61E-08</td>
<td>8.12E-11</td>
<td>1.05E+01</td>
<td>1.38E+02</td>
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<tr>
<td>$f_6$</td>
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<td>9.08E+01</td>
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<td>4.20E+01</td>
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<td>0.00E+00</td>
<td>0.00E+00</td>
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<tr>
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<td>4.57E+03</td>
<td>2.65E+03</td>
<td>3.29E+03</td>
<td>2.59E+03</td>
<td>3.23E+03</td>
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<td>4.67E+02</td>
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<tr>
<td>$f_{13}$</td>
<td>5.12E+14</td>
<td>5.29E+14</td>
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<td>3.70E+01</td>
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<td>3.88E+12</td>
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<tr>
<td>$f_{15}$</td>
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<td>2.85E+00</td>
<td>4.50E+00</td>
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<td>0.00E+00</td>
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<tr>
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<td>$f_{17}$</td>
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<td>1.11E+14</td>
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<td>$f_{18}$</td>
<td>1.07E+15</td>
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<td>$f_{22}$</td>
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</tr>
</tbody>
</table>

4. Experimental study

4.1. Benchmark functions and parameter settings

In order to verify the performance of the ALPSO algorithm, here we use 40 test functions from two groups in our experiments. The first group has 12 basic functions ($f_1$–$f_{12}$), which are shown in Table 1. The first five functions ($f_1$–$f_5$) are unimodal functions and the next seven functions are multimodal functions. By doing experiments on these functions, we can verify that if the ALPSO can maintain the fast convergence feature and have the ability of dealing with multimodal
functions. The second group has 28 functions (f13–f40) shown in Table 2 which are taken from the CEC2013 test suite. Details of these functions can be find in the report [12]. The first five functions (f13–f17) are unimodal functions, the next 15 functions (f18–f32) are multimodal functions and the last 8 functions (f33–f40) are composition functions. Every functions search space is complex, and we can testify the comprehensive performance of ALPSO by the experimental study on these functions.

The global optima of these functions shown in Table 1 are 0, the Domain is search range of every dimension.

In f8, the $y_i$ is update by $y_i = \begin{cases} x_i, & |x_i| \leq 0.5 \\ \text{round}(2x_i), & |x_i| \geq 0.5 \end{cases}$

In f11, the $y_i$ is update by $y_i = 1 + \frac{1}{2}(x_i + 1)$.
In $f_{11}$ and $f_{12}$, $u(x_j, a, k, m) = \begin{cases} k(x_j - a)m & x_j > a \\ 0 & -a \leq x_j \leq a \\ k(-x_j - 1)m & x_j < -a \end{cases}.$

To validate the performances of the ALPSO, five representative variants of PSO are chosen to compare with ALPSO. The first one is the global version PSO (GPSO), in the GPSO particles always learn from the $P_g$, so it has the feature of fast convergence when dealing with unimodal problem. The second one is the global version PSO which is designed to improve the swarm’s ability of global search, and the inertia weight is linearly decreased from 0.9 to 0.4. The third one is local version of PSO (LPSSO) using a ring topology. The fourth one is DMS-PSO, and the fifth one is CLPSO. The parameter settings of each algorithm are shown below in Table 3. The parameter $M$ and $R$ in DMS-PSO stands for the size of subswarm and regrouping period, respectively. The parameter $\gamma_M$ in CLPSO is the refreshing gap of a particle.

To make it fair when comparing experimental results of using different algorithms, each algorithm will run on the corresponding benchmark function independently 50 times and the mean error of results will be displayed in the tables of results comparison shown below. Here, the dimension of all benchmark functions $D$ is set as 30. The population size $N$ in each algorithm is set as 20. The maximum of the fitness evaluation $FEs$ number is $2 \times 10^5$. 

Fig. 4. Convergence process on 12 basic benchmark functions.
4.2. Results comparison on convergence accuracy

The comparison results on convergence accuracy including mean error (MEAN) and error’s standard deviation (SD) which are listed in Tables 4 and 5. Here, we mark the best results performed by those algorithms on each test functions with bold font, and mark the second best results of them with underlined.

As the value of MEAN shown in Tables 4 and 5, we can obviously see that GPSO performs well on some basic unimodal functions (f1–f3), but gets bad results when dealing with multimodal functions and the unimodal functions with complex search space on CEC2013 test suite. A possible reason is that, since the inertia weight is linearly decreased in GPSO, it helps GPSO adjust the balance between the exploration and exploitation and have a good convergence when dealing with unimodal problems; while for multimodal problems, the linearly decreased weight can’t adjust well along the evolving process and get good results. And for LPSO, it has the ability of maintaining the diversity of population by producing restrictions on global search, the structure of particles may not suit all test functions, so it can only performs well on some specific functions (i.e., f5, f9, f19).

As we know that CLPSO and DMS-PSO have a competitive capability on solving multimodal problems and they perform well on several test functions shown in Tables 4 and 5. Especially for DMS-PSO, it can get a second best results on several multimodal test functions, i.e., f4, f10, f18, f22, f26, f27, f34 and f40, it even gets results that superior to ALPSO on some multimodal test functions, i.e., f19, f24, f25 and f32. A possible reason may be that the competitive learning based strategy used in ALPSO wastes some fitness evolutions since the search direction of swarm needs to be constantly adjusted when swarm frequently gets trapped into local optima. On the contrary, as the results shown in Tables 4 and 5, DMS-PSO performs poorer on 5 basic unimodal functions and 5 unimodal test functions than ALPSO. The reason is that, DMS-PSO use a multi-swarm strategy to maintain the diversity of swarm within the entire search space, which has restriction on the convergence speed of DMS-PSO.

As mentioned above, ALPSO can jump out from local optima and get the ability of global search by TSDM and the self-learning based candidate generation strategy, which result in that swarm may adaptively adjust it’s search direction to a more excellent particle and stop falling into local optima especially when swarm searches in a complex space. Moreover, ALPSO can get the ability of local search by the mechanism of competitive learning based prediction strategy, which chooses $P_k$ with more leading ability. As the Tables 4 and 5 shows, ALPSO gets the most favorable results on 6 out of 7 basic multimodal test functions and gets the best or the second best results on 12 out of 15 CEC2013 multimodal test functions, except for f19, f24 and f25, which means ALPSO has an excellent performances when dealing with multimodal test functions. Therefore, the efficiency of ALPSO to deal with multimodal test functions is verified.

Simultaneously, from Tables 4 and 5, we can see that ALPSO is not restricted when dealing with simple unimodal test functions. The reason is that there is no need for swarm in ALPSO to frequently adjust the search direction as the swarm’s evolutionary state which is determined by TSDM is fine. In other words, the strategies utilized in ALPSO produce little impact on the algorithm’s convergence speed when algorithm solves simple unimodal test functions, which resulting in that ALPSO also performs well on all unimodal test functions described above (i.e., f1–f5, f13–f17). As for the results on
Fig. 6. Convergence process on 15 basic multimodal functions of CEC'2013.
composition functions (f33–f40) which are shown in Table 5, ALPSO gets the best results or the second best results on 6 out of 8 composition functions, which presents the performance of ALPSO to solve more tough problems.

In addition, as the value of SD shown in Tables 4 and 5, we can see that the SD's values of ALPSO on most functions (f5–f12, f13–f18, f21–f23, f26–f34, f37–f40) is much smaller than others, which means that the performance of our proposed ALPSO is steady in convergence process.

4.3. Results comparison on convergence speed

We then conduct experiments on the convergence speed of the same 40 benchmark test functions to further testify the convergence speed of ALPSO. Figs. 4–7 show the convergence process of the algorithms.

From the above figures, we can see that the proposed ALPSO can converge with an ideal convergence speed, especially on multimodal functions. And from the convergence process figures of the 12 basic test functions, it can also be find that ALPSO can converge to the global optimal value on the multimodal functions in the first group (from f5 to f12), and has the fastest convergence speed among these algorithms. This might because that, in ALPSO, once the swarm get trapped into a local optimum, the swarm will timely adjust its search direction by TSDM, and this mechanism help ALPSO jump out from local optima and accelerate the searching speed especially when ALPSO dealing with these multimodel functions within a complex search space. And from Fig. 5–7, we can also find that, ALPSO still have a fast convergence rate and the speed of convergence ranked first on many other functions in the second group (i.e. f13, f14, f16, f17, f18, f20, f21, f23, f26, f27, f30–f40).

From the above results, we can now conclude that ALPSO can get better results than the other five algorithms, not only on the solution accuracy but also the convergence speed, especially when dealing complex problems.

5. Conclusion and future work

In this paper, we present a hybrid PSO algorithm based on adaptive learning strategy, namely ALPSO to alleviate the premature convergence of PSO on many complex problems. We mainly focus on the improvement of the swarm structure
learning and particles’ local search strategy learning. As we know, the main reason that the basic PSO usually gets trapped in local optima is the particles in the population mostly learn from the global best. Here, we employ a self-learning based candidate generation strategy which generates a new candidate not only utilize the information from global best particle but also the historical individual best particles in all dimensions. This strategy contributes to the improvement on the exploration ability of our new algorithm. At the same time, we also propose a competitive learning based prediction strategy to help choose the best candidate in the evolution process, which can help to find those particles with potential ability to lead the swarm to find the global best and retain them in the population. As a result of this, the exploitation ability can also be improved. Furthermore, for the sake of getting a good balance between exploration and exploitation, we design a tolerance based search direction adjustment mechanism, which help decide the proper timing of the particles change the search direction in the solution space. We conduct the experimental study on 40 test functions, including 12 basic test functions, 20 test functions and 8 composition functions from CEC2013 Test Suite. The results of this ALPSO algorithm comparing with 5 state-of-the-art algorithms show it can get better convergence accuracy as well as faster convergence speed in most cases.

The experimental results show the competitive performance of ALPSO and the ability to solve complex problems such as multimodal test problems and composition test problems is verified. As the problems in reality are getting more and more complicated such as financial optimization problems and scheduling optimization problems, in the future, we will further go on the theoretical analysis of the parameter settings in ALPSO, and try to investigate the applications of this ALPSO algorithm into solving various challenging optimization problems in reality.

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References